5-3 Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $11x^6 - 5x^5 + 4x^2$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of the polynomial when written in standard form.
Degree = 6,
leading coefficient = 11

2. $-10x^7 - 5x^3 + 4x - 22$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of the polynomial when written in standard form.
Degree = 7,
leading coefficient = –10

3. $14x^4 - 9x^3 + 3x - 4y$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of the polynomial when written in standard form. This polynomial is not in one variable because there are two variables, x and y.

4. $8x^5 - 3x^2 + 4xy - 5$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of the polynomial when written in standard form. This polynomial is not in one variable because there are two variables, x and y.

Find $w(5)$ and $w(-4)$ for each function.

5. $w(x) = -2x^3 + 3x - 12$

SOLUTION:

$$\begin{aligned} w(5) &= -2(5)^3 + 3(5) - 12 \\ &= -250 + 15 - 12 \\ &= -247 \end{aligned}$$

$$\begin{aligned} w(-4) &= -2(-4)^3 + 3(-4) - 12 \\ &= 128 - 12 - 12 \\ &= 104 \end{aligned}$$

6. $w(x) = 2x^4 - 5x^3 + 3x^2 - 2x + 8$

SOLUTION:

$$\begin{aligned} w(5) &= 2(5)^4 - 5(5)^3 + 3(5)^2 - 2(5) + 8 \\ &= 1250 - 625 + 75 - 10 + 8 \\ &= 698 \end{aligned}$$

$$\begin{aligned} w(-4) &= 2(-4)^4 - 5(-4)^3 + 3(-4)^2 - 2(-4) + 8 \\ &= 512 + 320 + 48 + 8 + 8 \\ &= 896 \end{aligned}$$

If $c(x) = 4x^3 - 5x^2 + 2$ and $d(x) = 3x^2 + 6x - 10$, find each value.

7. $c(y^3)$

SOLUTION:

$c(y^3) = 4\left(y^3\right)^3 - 5\left(y^3\right)^2 + 2$ Substitute y^3 for x.

$= 4y^9 - 5y^6 + 2$ Simplify.

8. $-4[d(3z)]$

SOLUTION:

$-4[d(3z)] = -4\left[3(3z)^2 + 6(3z) - 10\right]$ Substitute 3z for x.

$= -4\left[27z^2 + 18z - 10\right]$ Simplify.

$= -108z^2 - 72z + 40$ Distributive Property

9. $6c(4a) + 2d(3a - 5)$

SOLUTION:

$6c(4a) + 2d(3a - 5)$

$= 6\left[4(4a)^3 - 5(4a)^2 + 2\right] + 2\left[3(3a-5)^2 + 6(3a-5) - 10\right]$ Substitute 4a for x in c(x) and 3a - 5 for x in

$= 6\left[256a^3 - 80a^2 + 2\right] + 2\left[3(9a^2 + 25 - 30a) + 18a - 30 - 10\right]$ Simplify.

$= 1536a^3 - 480a^2 + 12 + 2\left[27a^2 + 75 - 90a + 18a - 30 - 10\right]$ Simplify.

$= 1536a^3 - 480a^2 + 12 + 2\left[27a^2 - 72a + 35\right]$ Simplify.

$= 1536a^3 - 480a^2 + 12 + 54a^2 - 144a + 70$ Distributive Property

$= 1536a^3 - 426a^2 - 144a + 82$ Combine like terms.

10. $-3c(2b) + 6d(4b - 3)$

SOLUTION:

$$-3[c(2b)] + 6[d(4b-3)]$$

$= -3\left[4(2b)^3 - 5(2b)^2 + 2\right] + 6\left[3(4b-3)^2 + 6(4b-3) - 10\right]$ Substitute 2b for x in c(x0 adn 4b - 3 for x

$= -3\left[32b^3 - 20b^2 + 2\right] + 6\left[3(16b^2 - 24b + 9) + 24b - 18 - 10\right]$ Simplify.

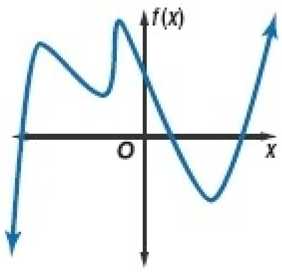
$= -96b^3 + 60b^2 - 6 + 6\left[48b^2 - 72b + 27 + 24b - 18 - 10\right]$ Simplify.

$= -96b^3 + 60b^2 - 6 + 6\left[48b^2 - 48b - 1\right]$ Simplify.

$= -96b^3 + 60b^2 - 6 + 288b^2 - 288b - 6$ Distributive Property

$= -96b^3 + 348b^2 - 288b - 12$ Combine like terms.

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeros.

11. 

SOLUTION:

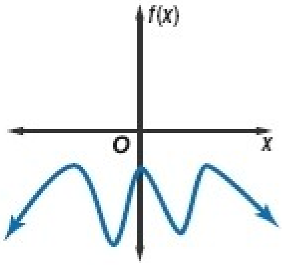
a. As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach positive infinity.

$f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to +\infty$ as $x \to +\infty$.

The end behavior is in opposite directions.

b. Since the end behavior is in opposite directions, it is an odd-degree function.

c. The graph intersects the x-axis at three points, so there are three real zeros.



12.

SOLUTION:

a. As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -\infty$ as $x \to +\infty$.

The end behavior is in the same directions.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at zero points, so there are no real zeros.

CCSS PERSEVERANCE State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

13. $-6x^6 - 4x^5 + 13xy$

SOLUTION:

not a polynomial in one variable because there are two variables, x and y.

14. $3a^7 - 4a^4 + \frac{3}{a}$

SOLUTION:

not a polynomial in one variable; $\frac{3}{a}$ has the variable with an exponent less than 0.

15. $8x^5 - 12x^6 + 14x^3 - 9$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 6, leading coefficient = –12.

16. $-12 - 8x^2 + 5x - 21x^7$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 7, leading coefficient = –21.

17. $15x - 4x^3 + 3x^2 - 5x^4$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 4, leading coefficient = –5.

18. $13b^3 - 9b + 3b^5 - 18$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 5, leading coefficient = 3.

19. $(d + 5)(3d - 4)$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 2, leading coefficient = 3.

20. $(5 - 2y)(4 + 3y)$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 2, leading coefficient = –6.

21. $6x^5 - 5x^4 + 2x^9 - 3x^2$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 9, leading coefficient = 2.

22. $7x^4 + 3x^7 - 2x^8 + 7$

SOLUTION:

The degree of the polynomial is the value of the greatest exponent. The leading coefficient is the coefficient of the first term of a polynomial written in standard form. The degree = 8, leading coefficient = –2.

Find *p* (–6) and *p* (3) for each function.

23. $p(x) = x^4 - 2x^2 + 3$

SOLUTION:

$$\begin{aligned} p(-6) &= (-6)^4 - 2(-6)^2 + 3 \\ &= 1296 - 72 + 3 \\ &= 1227 \end{aligned}$$

$$\begin{aligned} p(3) &= (3)^4 - 2(3)^2 + 3 \\ &= 81 - 18 + 3 \\ &= 66 \end{aligned}$$

24. $p(x) = -3x^3 - 2x^2 + 4x - 6$

SOLUTION:

$$\begin{aligned} p(-6) &= -3(-6)^3 - 2(-6)^2 + 4(-6) - 6 \\ &= 648 - 72 - 24 - 6 \\ &= 546 \end{aligned}$$

$$\begin{aligned} p(3) &= -3(3)^3 - 2(3)^2 + 4(3) - 6 \\ &= -81 - 18 + 12 - 6 \\ &= -93 \end{aligned}$$

25. $p(x) = 2x^3 + 6x^2 - 10x$

SOLUTION:

$$\begin{aligned} P(-6) &= 2(-6)^3 + 6(-6)^2 - 10(-6) \\ &= -432 + 216 + 60 \\ &= -156 \end{aligned}$$

$$\begin{aligned} P(3) &= 2(3)^3 + 6(3)^2 - 10(3) \\ &= 54 + 54 - 30 \\ &= 78 \end{aligned}$$

26. $p(x) = x^4 - 4x^3 + 3x^2 - 5x + 24$

SOLUTION:

$$\begin{aligned} p(-6) &= (-6)^4 - 4(-6)^3 + 3(-6)^2 - 5(-6) + 24 \\ &= 1296 + 864 + 108 + 30 + 24 \\ &= 2322 \end{aligned}$$

$$\begin{aligned} p(3) &= (3)^4 - 4(3)^3 + 3(3)^2 - 5(3) + 24 \\ &= 81 - 108 + 27 - 15 + 24 \\ &= 9 \end{aligned}$$

27. $p(x) = -x^3 + 3x^2 - 5$

SOLUTION:

$$\begin{aligned} P(-6) &= -(-6)^3 + 3(-6)^2 - 5 \\ &= 216 + 108 - 5 \\ &= 319 \end{aligned}$$

$$\begin{aligned} P(3) &= -(3)^3 + 3(3)^2 - 5 \\ &= -27 + 27 - 5 \\ &= -5 \end{aligned}$$

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28. $p(x) = 2x^4 + x^3 - 4x^2$

SOLUTION:

$$\begin{aligned}P(-6) &= 2(-6)^4 + (-6)^3 - 4(-6)^2\\ &= 2592 - 216 - 144\\ &= 2232\\ P(3) &= 2(3)^4 + (3)^3 - 4(3)^2\\ &= 162 + 27 - 36\\ &= 153\end{aligned}$$

If $c(x) = 2x^2 - 4x + 3$ and $d(x) = -x^3 + x + 1$, find each value.

29. $c(3a)$

SOLUTION:

$$\begin{aligned}c(3a) &= 2(3a)^2 - 4(3a) + 3\\ &= 18a^2 - 12a + 3\end{aligned}$$

30. $5d(2a)$

SOLUTION:

$$\begin{aligned}5d(2a) &= 5\left[-(2a)^3 + (2a) + 1\right]\\ &= 5\left[-8a^3 + 2a + 1\right]\\ &= -40a^3 + 10a + 5\end{aligned}$$

31. $c(b^2)$

SOLUTION:

$$\begin{aligned}c(b^2) &= 2(b^2)^2 - 4(b^2) + 3\\ &= 2b^4 - 4b^2 + 3\end{aligned}$$

32. $d(4a^2)$

SOLUTION:

$$\begin{aligned}d\left(4a^2\right) &= -(4a^2)^3 + (4a^2) + 1\\ &= -64a^6 + 4a^2 + 1\end{aligned}$$

33. $d(4y - 3)$

SOLUTION:

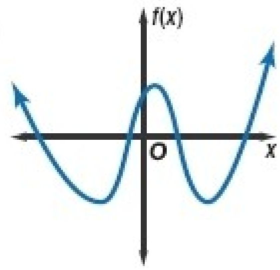
$$\begin{aligned}d\left(4y-3\right) &= -(4y-3)^3 + (4y-3) + 1\\ &= -\left[64y^3 + 3(4y)^2(-3) + 3(4y)(-3)^2 + (-3)^3\right] + 4y - 3 + 1\\ &= -64y^3 + 144y^2 - 108y + 27 + 4y - 2\\ &= -64y^3 + 144y^2 - 104y + 25\end{aligned}$$

34. $c(y^2 - 1)$

SOLUTION:

$$\begin{aligned} c(y^2-1) &= 2(y^2-1)^2 - 4(y^2-1)+3 \\ &= 2(y^4-2y^2+1)-4y^2+4+3 \\ &= 2y^4-4y^2+2-4y^2+7 \\ &= 2y^4-8y^2+9 \end{aligned}$$

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeros.

35. 

SOLUTION:

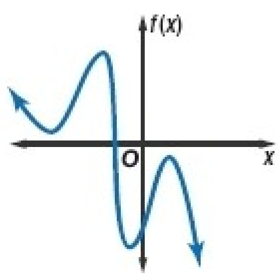
a. As the x-values approach negative infinity, the y-values approach positive infinity. As the x-values approach positive infinity, the y-values approach positive infinity.

$f(x) \to +\infty$ *as* $x \to -\infty$, $f(x) \to +\infty$ *as* $x \to +\infty$.

The end behavior is in the same direction.

b. Since the end behavior is in same direction, it is an even-degree function.

c. The graph intersects the x-axis at four points, so there are four real zeros.

36. 

SOLUTION:

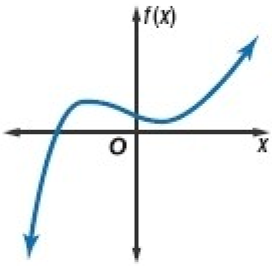
a. As the x-values approach negative infinity, the y-values approach positive infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to +\infty$ *as* $x \to -\infty$, $f(x) \to -\infty$ *as* $x \to +\infty$.

The end behavior is in opposite direction.

b. Since the end behavior is in opposite direction, it is an odd-degree function.

c. The graph intersects the x-axis at one point, so there is one real zero.

37. 

SOLUTION:

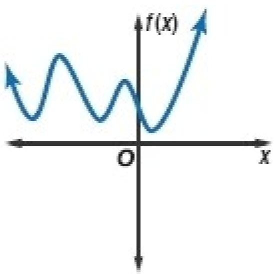
a. As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach positive infinity.

$f(x) \to -\infty$ *as* $x \to -\infty$, $f(x) \to +\infty$ *as* $x \to +\infty$.

The end behavior is in opposite direction.

b. Since the end behavior is in opposite direction, it is an odd-degree function.

c. The graph intersects the x-axis at one point, so there is one real zero.

38. 

SOLUTION:

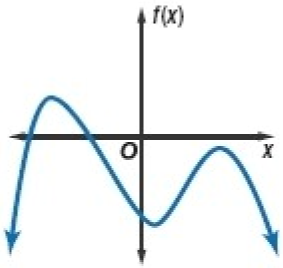
a. As the x-values approach negative infinity, the y-values approach positive infinity. As the x-values approach positive infinity, the y-values approach positive infinity.

$f(x) \to +\infty$ *as* $x \to -\infty$. $f(x) \to +\infty$ *as* $x \to +\infty$.

The end behavior is in same directions.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at zero points, so there are no real zeros.

39. 

SOLUTION:

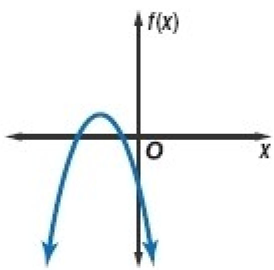
a. As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to -\infty$ as $x \to -\infty$. $f(x) \to -\infty$ as $x \to +\infty$.

The end behavior is in the same direction.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at two points, so there are two real zeros.

40. 

SOLUTION:

a. As the x-values approach negative infinity, the y-values approach positive infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to -\infty as x \to -\infty$. $f(x) \to -\infty as x \to +\infty$.

The end behavior is in the same direction.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at two points, so there are two real zeros.

41. **PHYSICS** For a moving object with mass m in kilograms, the kinetic energy KE in joules is given by the function $KE(v) = 0.5mv^2$, where v represents the speed of the object in meters per second. Find the kinetic energy of an all-terrain vehicle with a mass of 171 kilograms moving at a speed of 11 meters/second.

SOLUTION:

$$\begin{aligned} KE(v) &= 0.5(171)(11^2) \\ &= 10345.5 \text{ joules} \end{aligned}$$

42. **CCSS MODELING** A microwave manufacturing firm has determined that their profit function is $P(x) = -0.0014x^3 + 0.3x^2 + 6x - 355$ where x is the number of microwaves sold annually.

a. Graph the profit function using a calculator.
b. Determine a reasonable viewing window for the function.
c. Approximate all of the zeros of the function using the **CALC** menu.
d. What must be the range of microwaves sold in order for the firm to have a profit?

SOLUTION:

a.

b. Sample answer: [–500, 500] scl: 50 by [–10,000, 10,000] scl: 1000
c. – 41.0, 27.1, 228.2
d. 28 to 228 microwaves

Find p (–2) and p (8) for each function.

43. $p(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 4x^2$

SOLUTION:

$$\begin{aligned} P(-2) &= \frac{1}{4}(-2)^4 + \frac{1}{2}(-2)^3 - 4(-2)^2 \\ &= 4 - 4 - 16 \\ &= -16 \end{aligned}$$

$$\begin{aligned} P(8) &= \frac{1}{4}(8)^4 + \frac{1}{2}(8)^3 - 4(8)^2 \\ &= 1024 + 256 - 256 \\ &= 1024 \end{aligned}$$

44. $p(x)=\frac{1}{8}x^4-\frac{3}{2}x^3+12x-18$

SOLUTION:

$$\begin{aligned} p(-2) &= \frac{1}{8}(-2)^4-\frac{3}{2}(-2)^3+12(-2)-18 \\ &= 2+12-24-18 \\ &= -28 \end{aligned}$$

$$\begin{aligned} p(8) &= \frac{1}{8}(8)^4-\frac{3}{2}(8)^3+12(8)-18 \\ &= 512-768+96-18 \\ &= -178 \end{aligned}$$

45. $p(x)=\frac{3}{4}x^4-\frac{1}{8}x^2+6x$

SOLUTION:

$$\begin{aligned} p(-2) &= \frac{3}{4}(-2)^4-\frac{1}{8}(-2)^2+6(-2) \\ &= 12-0.5-12 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} p(8) &= \frac{3}{4}(8)^4-\frac{1}{8}(8)^2+6(8) \\ &= 3072-8+48 \\ &= 3112 \end{aligned}$$

46. $p(x)=\frac{5}{8}x^3-\frac{1}{2}x^2+\frac{3}{4}x+10$

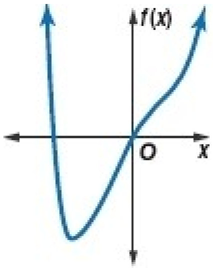
SOLUTION:

$$\begin{aligned} p(-2) &= \frac{5}{8}(-2)^3-\frac{1}{2}(-2)^2+\frac{3}{4}(-2)+10 \\ &= -5-2-1.5+10 \\ &= 1.5 \end{aligned}$$

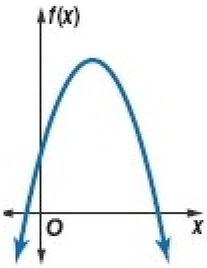
$$\begin{aligned} p(8) &= \frac{5}{8}(8)^3-\frac{1}{2}(8)^2+\frac{3}{4}(8)+10 \\ &= 320-32+6+10 \\ &= 304 \end{aligned}$$

Use the degree and end behavior to match each polynomial to its graph.

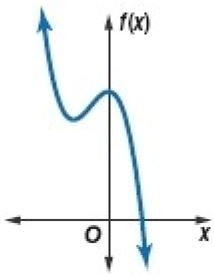
A.



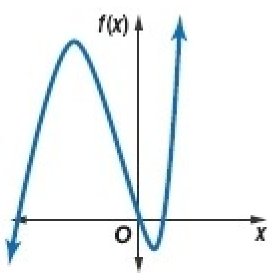
B.



C.



D.



47. $f(x) = x^3 + 3x^2 - 4x$

SOLUTION:

Since the degree of the polynomial is 3, the graph must have 2 turning points. And the leading coefficient of the graph is positive. Therefore, the correct choice is D.

48. $f(x) = -2x^2 + 8x + 5$

SOLUTION:

Since the degree of the polynomial is 2, the graph must have 1 turning point. And the function is an even degree function. Therefore, the end behavior of the graph must be in the same direction. The correct choice is B.

49. $f(x) = x^4 - 3x^2 + 6x$

SOLUTION:

Since the degree of the polynomial is 4, the graph must have 3 turning points. And the function is an even degree function. Therefore, the end behavior of the graph must be in the same direction. The correct choice is A.

50. $f(x) = -4x^3 - 4x^2 + 8$

SOLUTION:

Since the degree of the polynomial is 3, the graph must have 2 turning point2. And the leading coefficient of the graph is negative. Therefore, the graph opens down. The correct choice is B.

If $c(x) = x^3 - 2x$ and $d(x) = 4x^2 - 6x + 8$, find each value.

51. $3c(a - 4) + 3d(a + 5)$

SOLUTION:

$$3c(a-4)+3d(a+5)$$

$=3\left[(a-4)^3-2(a-4)\right]+3\left[4(a+5)^2-6(a+5)+8\right]$ Substitute a - 4 for x in c(x)

$=3\left[a^4-3a^2(4)+3a(-4)^2-64-2a+8\right]+3\left[4(a^2+10a+25)-6a-30+8\right]$ Simplify.

$=3\left[a^4-12a^2+48a-56-2a\right]+3\left[4a^2+40a+100-6a-22\right]$ Simplify.

$=3\left[a^4-12a^2+46a-56\right]+3\left[4a^2+34a+78\right]$ Simplify.

$=3a^4-36a^2+138a-168+12a^2+102a+234$ Distributive Property

$=3a^4-24a^2+240a+66$ Combine like terms.

52. $-2d(2a + 3) - 4c(a^2 + 1)$

SOLUTION:

$$-2d(2a+3)-4c(a^2+1)$$

$=-2\left[4(2a+3)^2-6(2a+3)+8\right]-4\left[(a^2+1)^3-2(a^2+1)\right]$ Substitute $2a+3$ for x in $d(x)$ and c

$=-2\left[4(4a^2+12a+9)-12a-18+8\right]-4\left[a^6+1+3a^4+3a^2-2a^2-2\right]$ Simplify.

$=-2\left[16a^2+48a+36-12a-10\right]-4\left[a^6+3a^4+3a^2-2a^2-1\right]$ Simplify.

$=-2\left[16a^2+36a+26\right]-4\left[a^6+3a^4+a^2-1\right]$ Simplify.

$=-32a^2-72a-52-4a^6-12a^4-4a^2+4$ Distributive Property

$=-4a^6-12a^4-36a^2-72a-48$ Combine like terms.

53. $5c(a^2) - 8d(6 - 3a)$

SOLUTION:

$$5c(a^2) - 8d(6-3a)$$

$= 5\left[\left(a^2\right)^3 - 2(a^2)\right] - 8\left[4(6-3a)^2 - 6(6-3a) + 8\right]$ Substitute a^2 for x in $c(x)$ and $6 - 3a$ for x in $d(x)$.

$= 5\left[a^6 - 2a^2\right] - 8\left[4(36 - 36a + 9a^2) - 36 + 18a + 8\right]$ Simplify.

$= 5a^6 - 10a^2 - 8\left[4(36 - 36a + 9a^2) - 36 + 18a + 8\right]$ Simplify.

$= 5a^6 - 10a^2 - 8\left[144 - 144a + 36a^2 - 28 + 18a\right]$ Simplify.

$= 5a^6 - 10a^2 - 1152 + 1152a + 224 - 144a$ Distributive Property

$= 5a^6 - 298a^2 + 1008a - 928$ Combine like terms.

54. $-7d(a^3) + 6c(a^4 + 1)$

SOLUTION:

$$-7d(a^3) + 6c(a^4+1)$$

$= -7\left[4\left(a^3\right)^2 - 6(a^3) + 8\right] + 6\left[(a^4+1)^3 - 2(a^4+1)\right]$ Substitute a^3 for x in $d(x)$ and $a^4 + 1$ for x in $c(x)$

$= -7\left[4a^6 - 6a^3 + 8\right] + 6\left[a^{12} + 3a^8 + 3a^4 + 1 - 2a^4 - 2\right]$ Simplify.

$= -7\left[4a^6 - 6a^3 + 8\right] + 6\left[a^{12} + 3a^8 + a^4 - 1\right]$ Simplify.

$= -28a^6 + 42a^3 - 56 + 6a^{12} + 18a^8 + 6a^4 - 6$ Distributive Property

$= 6a^{12} + 18a^8 - 28a^6 + 6a^4 + 42a^3 - 62$ Combine like terms.

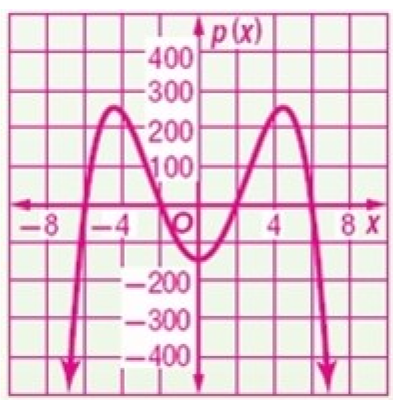
55. **BUSINESS** A clothing manufacturer's profitability can be modeled by $p(x) = -x^4 + 40x^2 - 144$, where x is the number of items sold in thousands and $p(x)$ is the company's profit in thousands of dollars.
a. Use a table of values to sketch the function.
b. Determine the zeros of the function.
c. Between what two values should the company sell in order to be profitable?
d. Explain why only two of the zeros are considered in part c.

SOLUTION:

a. Table:

x	$p(x)$
–7	–585
–6	0
–4	240
–3	135
–2	0
0	–144
1	–105
2	0
4	240
6	0
7	–585

Plot the points on the coordinate plane. And connect them by a smooth curve.



b. –6, –2, 2, 6
c. The graph lies in the Quadrant I (both x and y values are positive) between $x = 2$ and $x = 6$. Therefore, the company should sell 2000 to 6000 items.
d. Sample answer: The negative values should not be considered because the company will not produce negative items.

56. **MULTIPLE REPRESENTATIONS** Consider $g(x) = (x - 2)(x + 1)(x - 3)(x + 4)$

a. ANALYTICAL Determine the x- and y-intercepts, roots, degree, and end behavior of $g(x)$.

b. ALGEBRAIC Write the function in standard form.

c. TABULAR Make a table of values for the function.

d. GRAPHICAL Sketch a graph of the function by plotting points and connecting them with a smooth curve.

SOLUTION:

a. Degree: 4;

$(x-2)(x+1)(x-3)(x+4)=0$ Original equation

$x=2$ or $x=-1$ or $x=3$ or $x=-4$ Solve for x.

x-intercepts: $x = 2, -1, 3, -4$.
y-intercept: $y = 24$.
End behavior: as $x \to -\infty, g(x) \to +\infty$; as $x \to +\infty, g(x) \to +\infty$

b.

$g(x)=(x-2)(x+1)(x-3)(x+4)$ Original equation

$=(x^2-x-2)(x^2+x-12)$ FOIL

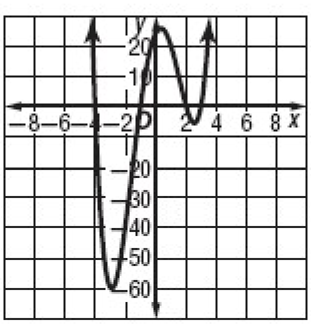
$=x^4+x^3-12x^2-x^3-x^2+12x-2x^2-2x+24$ Distributive Property

$=x^4-15x^2+10x+24$ Combine like terms.

c.

x	g(x)
–5	224
–4	0
–3	–60
–2	–40
–1	0
0	24
1	20
2	0
3	0
4	80
5	324

d.



5-3 Polynomial Functions

Describe the end behavior of the graph of each function.

57. $f(x) = -5x^4 + 3x^2$

SOLUTION:

As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$.

58. $g(x) = 2x^5 + 6x^4$

SOLUTION:

As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach positive infinity.

$g(x) \to -\infty$ as $x \to -\infty$; $g(x) \to +\infty$ as $x \to +\infty$.

59. $h(x) = -4x^7 + 8x^6 - 4x$

SOLUTION:

As the x-values approach negative infinity, the y-values approach positive infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$h(x) \to +\infty$ as $x \to -\infty$; $h(x) \to -\infty$ as $x \to +\infty$.

60. $f(x) = 6x - 7x^2$

SOLUTION:

As the x-values approach negative infinity, the y-values approach negative infinity. As the x-values approach positive infinity, the y-values approach negative infinity.

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$.

61. $g(x) = 8x^4 + 5x^5$

SOLUTION:

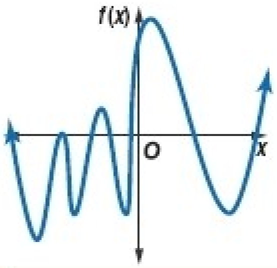
$g(x) \to -\infty$ as $x \to -\infty$; $g(x) \to +\infty$ as $x \to +\infty$.

62. $h(x) = 9x^6 - 5x^7 + 3x^2$

SOLUTION:

$h(x) \to +\infty$ as $x \to -\infty$; $h(x) \to -\infty$ as $x \to +\infty$.

63. **CCSS CRITIQUE** Shenequa and Virginia are determining the number of real zeros of the graph. Is either of them correct? Explain your reasoning.



Shenequa

There are 7 real zeros because the graph intersects the x-axis 7 times.

Virginia

There are 8 real zeros because the graph intersects the x-axis 7 times, and there is a double zero.

SOLUTION:

A double zero still represents 1 real zero. You will learn in a later lesson that a double zero is an indication of a multiple linear factor. Since the graph intersects the x-axis 7 times, the polynomial will have exactly 7 real zeros. Therefore, Shenequa is correct.

64. **CHALLENGE** Of $f(x)$ and $g(x)$, which function has more potential real zeros? What is the degree of that function?

x	−24	−18	−12	−6	0	6	12	18	24
$f(x)$	−8	−1	3	−2	4	7	−1	−8	5

$g(x) = x^4 + x^3 - 13x^2 + x + 4$

SOLUTION:

The sign of $f(x)$ changes for 5 times. So, $f(x)$ has the potential for 5 or more real zeros and a degree of 5 or more. Since $g(x)$ has a degree of 4, it has the potential for 4 real zeros.

So, $f(x)$ has more potential real zeros.

65. **CHALLENGE** If $f(x)$ has a degree of 5 and a positive leading coefficient and $g(x)$ has a degree of 3 and a positive leading coefficient, determine the end behavior of $\frac{f(x)}{g(x)}$. Explain your reasoning.

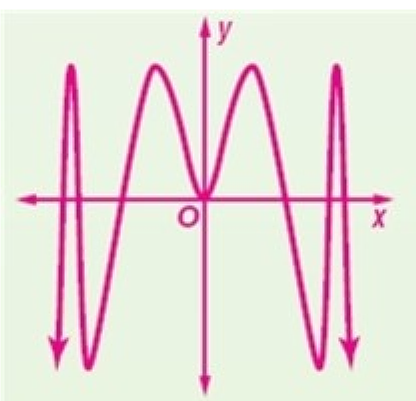
SOLUTION:
Sample answer:
Since $f(x)$ has a degree of 5 and $g(x)$ has a degree of 3, then $\frac{f(x)}{g(x)}$ will have a degree of 2. Since both functions have positive leading coefficients, their quotient will also have a positive leading coefficient. Therefore, the end behavior is:

$$f(x) \to +\infty \ \ as \ \ x \to -\infty;\ f(x) \to +\infty \ \ as \ \ x \to +\infty;$$

$\frac{f(x)}{g(x)}$ will become a 2nd-degree function with a positive leading coefficient.

66. **OPEN ENDED** Sketch the graph of an even-degree polynomial with 7 real zeros, one of them a double zero.

SOLUTION:
Sample answer:
An even degree polynomial function must go the same direction on both ends. Draw a graph that crosses the x-axis in three places to the left of the origin and three places to the right of the origin and goes towards $-\infty$ on both ends. Draw the graph so it is tangent to the x-axis at the origin. Since the graph has 7 x-intercepts, the polynomial has 7 real zeros with a double zero at the origin.



67. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.
A polynomial function that has four real zeros is a fourth-degree polynomial.

SOLUTION:
Sometimes; a polynomial function with four real zeros may be a sixth-degree polynomial function with two imaginary zeros. A polynomial function that has four real zeros is at least a fourth-degree polynomial.

68. **WRITING IN MATH** Describe what the end behavior of a polynomial function is and how to determine it.

SOLUTION:
Sample answer: The end behavior of a polynomial function is what the graph does as the input value approaches negative and positive infinity. It can be determined by the leading coefficient and the degree of the polynomial.

69. **SHORT RESPONSE** Four students solved the same math problem. Each student's work is shown below. Who is correct?

Student A

$$x^2x^{-5} = \frac{x^2}{x^5} = \frac{1}{x^3}, x \neq 0$$

Student B

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^{-7}, x \neq 0$$

Student C

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^7, x \neq 0$$

Student D

$$x^2x^{-5} = \frac{x^2}{x^5} = x^3, x \neq 0$$

SOLUTION:

$$a^{-m} = \frac{1}{a^m}, a \neq 0; \quad \text{Definition of negative exponents}$$

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0; \quad \text{Quotient of Powers}$$

Student A is correct.

70. **SAT/ACT** What is the remainder when $x^3 - 7x + 5$ is divided by $x + 3$?

A –11
B –1
C 1
D 11
E 35

SOLUTION:

Use synthetic division to divide.

$$\begin{array}{r|rrrr} \underline{-3} & 1 & 0 & -7 & 5 \\ & & -3 & +9 & -6 \\ \hline & 1 & -3 & +2 & |-1 \end{array}$$

The remainder is –1.
So, the correct choice is B.

71. **EXTENDED RESPONSE** A company manufactures tables and chairs. It costs $40 to make each table and $25 to make each chair. There is $1440 available to spend on manufacturing each week. Let t = the number of tables produced and c = the number of chairs produced.

a. The manufacturing equation is $40t + 25c = 1500$. Construct a graph of this equation.

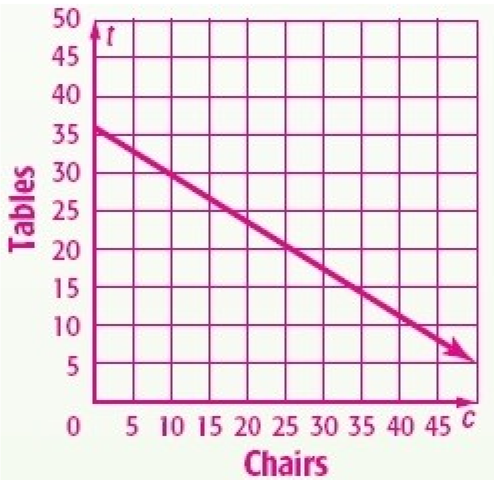
b. The company always produces two chairs with each table. Write and graph an equation to represent this situation on the same graph as the one in part **a**.

c. Determine the number of tables and chairs that the company can produce each week.

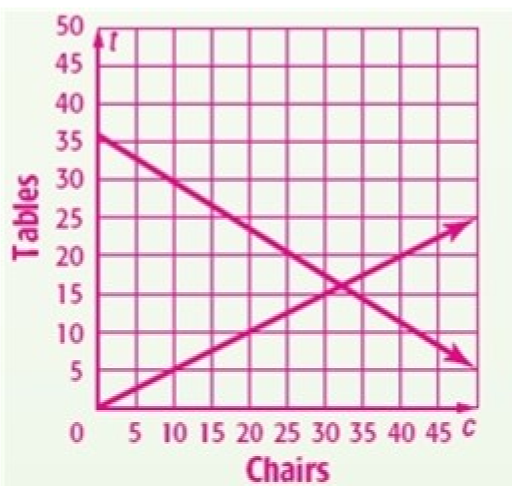
d. Explain how to determine this answer using the graph.

SOLUTION:

a. Graph the equation.



b. Since the company produces two chairs with each table, the equation should be $2t = c$.
That is, $t = 0.5c$.
Graph this equation on the same coordinate plane.



c. The graphs are intersecting at the point (32, 16). Therefore, 32 chairs and 16 tables can be produced each week.

d. Sample answer: This can be determined by the intersection of the graphs. This point of intersection is the optimal amount of tables and chairs manufactured.

72. If $i = \sqrt{-1}$ then $5i(7i) =$

F 70
H –35
G 35
J –70

SOLUTION:

$$\begin{aligned} 5i(7i) &= 35i^2 && \text{Multiply.} \\ &= 35\left(\sqrt{-1}\right)^2 && \text{Substitute } i = \sqrt{-1}. \\ &= 35(-1) && \text{Simplify.} \\ &= -35 && \text{Multiply.} \end{aligned}$$

The correct choice is H.

Simplify.

73. $\dfrac{16x^4y^3 + 32x^6y^5z^2}{8x^2y}$

SOLUTION:

$$\begin{aligned} &\frac{16x^4y^3 + 32x^6y^5z^2}{8x^2y} \\ &= \frac{16x^4y^3}{8x^2y} + \frac{32x^6y^5z^2}{8x^2y} && \text{Sum of quotients} \\ &= 2x^2y^2 + 4x^4y^4z^2 && \text{Divide.} \end{aligned}$$

74. $\dfrac{18ab^4c^5 - 30a^4b^3c^2 + 12a^5bc^3}{6abc^2}$

SOLUTION:

$$\begin{aligned} &\frac{18ab^4c^5 - 30a^4b^3c^2 + 12a^5bc^3}{6abc^2} \\ &= \frac{18ab^4c^5}{6abc^2} - \frac{30a^4b^3c^2}{6abc^2} + \frac{12a^5bc^3}{6abc^2} && \text{Sum of quotients} \\ &= 3b^3c^3 - 5a^3b^2 + 2a^4c && \text{Divide.} \end{aligned}$$

75. $\dfrac{18c^5d^2 - 3c^2d^2 + 12a^5c^3d^4}{3c^2d^2}$

SOLUTION:

$$\frac{18c^5d^2 - 3c^2d^2 + 12a^5c^3d^4}{3c^2d^2}$$

$$= \frac{18c^5d^2}{3c^2d^2} - \frac{3c^2d^2}{3c^2d^2} + \frac{12a^5c^3d^4}{3c^2d^2} \quad \text{Sum of quotients}$$

$$= 6c^3 - 1 + 4a^5cd^2 \quad \text{Divide.}$$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

76. $8x^2 + 5xy^3 - 6x + 4$

SOLUTION:

Yes. A polynomial is an expression consisting of monomials in which the exponents are nonnegative integers. The degree of a polynomial is the degree of the monomial with the greatest degree. The monomial with the greatest degree is $5xy^3$. The degree of the polynomial is 4.

77. $9x^4 + 12x^6 - 16$

SOLUTION:

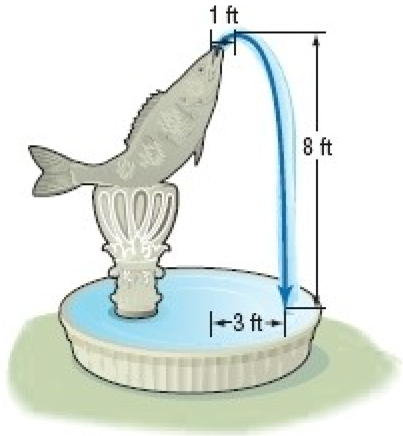
Yes. A polynomial is an expression consisting of monomials in which the exponents are nonnegative integers. The degree of a polynomial is the degree of the monomial with the greatest degree. The monomial with the greatest degree is $12x^6$. The degree of the polynomial is 6.

78. $3x^4 + 2x^2 - x^{-1}$

SOLUTION:

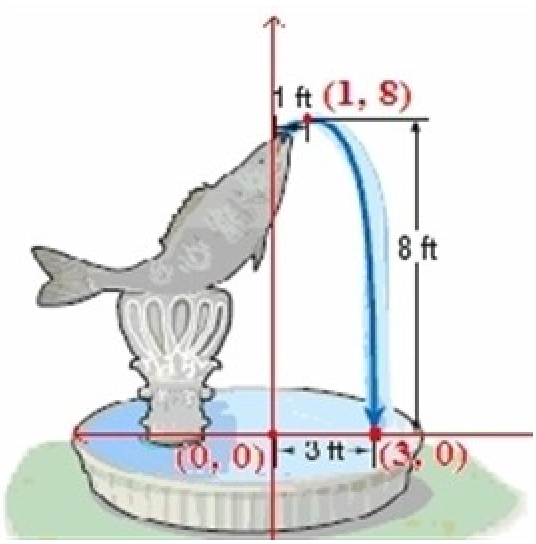
This is not a polynomial since one term has a variable with the exponent less than 0.

79. **FOUNTAINS** The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet.
a. If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet. Then compare the graph of the function to the parent function.
b. Suppose a worker increases the water pressure so that the stream reaches a maximum height of 12.5 feet at a distance of 15 inches from the jet. The water now lands 3.75 feet from the jet. Write a new quadratic function for $h(d)$. How do the changes in h and k affect the shape of the graph?



SOLUTION:

a. Find the vertex of the parabola.



From the graph the vertex of the parabola is (1, 8). Therefore, use the vertex form of the equation of the parabola.

$$h(d) = a(d-1)^2 + 8$$

The graph passes through the point (3, 0). So, substitute the point (3, 0) for $(d, h(d))$ in the equation to find a.

$$0 = a(3-1)^2 + 8$$
$$a = -2$$

Therefore, the function is:

$$\begin{aligned} h(d) &= -2(d-1)^2 + 8 \\ &= -2d^2 - 2 + 4d + 8 \\ &= -2d^2 + 4d + 6 \end{aligned}$$

The graph opens downward and is narrower than the parent graph, and the vertex is at (1, 8).

b. Here, the vertex of the new quadratic function is (15 in., 12.5 ft). That is (1.25ft, 12.5ft). The equation of the new quadratic function is:

$$h(d) = -2(d - 1.25)^2 + 12.5$$

It shifted the graph up 4.5 ft and to the right 3 in.

Solve each inequality.

80. $|2x + 4| \le 8$

SOLUTION:

$-8 \le 2x + 4 \le 8$	Rewrite the inequality.
$-8 - 4 \le 2x \le 8 - 4$	Subtract 4 from each side.
$-12 \le 2x \le 4$	Simplify.
$-6 \le x \le 2$	Divide each side by 2.

81. $|-3x + 2| \ge 4$

SOLUTION:

$-4 \ge -3x + 2 \ge 4$	Rewrite the inequality
$-4 - 2 \ge -3x \ge 4 - 2$	Subtract 2 from each side.
$-6 \ge -3x \ge 2$	Simplify.
$\frac{-6}{-3} \le x \le \frac{2}{-3}$	Divide each side by -3.
$2 \le x \le -\frac{2}{3}$	Simplify.
$x \le -\frac{2}{3}$ or $x \ge 2$	Rewrite inequality.

82. $|2x - 8| - 4 \le -6$

SOLUTION:

$\lvert 2x - 8\rvert - 4 \le -6$	Original inequality
$\lvert 2x - 8\rvert \le -6 + 4$	Add 4 to each side.
$\lvert 2x - 8\rvert \le -2$	Simplify.

Since the absolute value can not be less than a negative value, the inequality has no solution.

Determine whether each function has a maximum or minimum value, and find that value.

83. $f(x) = 3x^2 - 8x + 4$

SOLUTION:

For this function $a = 3 > 0$. Therefore, the graph opens up and the function has a minimum value.
The minimum value of the function is the y-coordinate of the vertex.
The x-coordinate of the vertex is:

$$\begin{aligned} x &= -\frac{-8}{2(3)} \\ &= \frac{4}{3} \end{aligned}$$

The y-coordinate of the vertex is:

$$\begin{aligned} f\left(\frac{4}{3}\right) &= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 \\ &= -\frac{4}{3} \end{aligned}$$

The minimum value of the function is $-\frac{4}{3}$.

84. $f(x) = -4x^2 + 2x - 10$

SOLUTION:

For this function $a = -4 < 0$. Therefore, the graph opens down and the function has a maximum value.
The maximum value of the function is the y-coordinate of the vertex.
The x-coordinate of the vertex is:

$$\begin{aligned} x &= -\frac{2}{2(-4)} \\ &= 0.25 \end{aligned}$$

The y-coordinate of the vertex is:

$$\begin{aligned} f(0.25) &= -4(0.25)^2 + 2(0.25) - 10 \\ &= -9.75 \end{aligned}$$

The maximum value of the function is –9.75.

85. $f(x) = -0.25x^2 + 4x - 5$

SOLUTION:

For this function $a = -0.25 < 0$. Therefore, the graph opens down and the function has a maximum value.
The maximum value of the function is the y-coordinate of the vertex.
The x-coordinate of the vertex is:

$$\begin{aligned} x &= -\frac{4}{2(-0.25)} \\ &= 8 \end{aligned}$$

The y-coordinate of the vertex is:

$$\begin{aligned} f(0.25) &= -0.25(8)^2 + 4(8) - 5 \\ &= 11 \end{aligned}$$

The maximum value of the function is 11.